Influence of intrinsic decoherence on nonclassical properties of the output of a Bose-Einstein condensate

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Abstract

We investigate nonclassical properties of the output of a Bose-Einstein condensate in Milburn's model of intrinsic decoherence. It is shown that the squeezing property of the atom laser is suppressed due to decoherence. Nevertheless, if some very special conditions were satisfied, the squeezing properties of atom laser could be robust against the decoherence.

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I. INTRODUCTION

Recently, the development of dilute gas Bose-Einstein condensation (BEC) [1,2,3] has opened up the study of atom laser [4-7], which is the matter wave analogs of optical lasers. The ideal atom laser beam is a single frequency de Broglie wave with well-defined intensity and phase. The first realization of a pulsed atom laser was achieved with sodium atoms at MIT by coupling a BEC from a magnetically trapped state to an untrapped state using a rf pulse [8]. Then, it was repeated with long rf pulses [9] and Raman transitions [10].

In recent years, much attention has been focused on the problem of nonlinear atomic optics. The four matter-wave mixing was realized in the remarkable experiment [11] by making use of the optical technique of Bragg diffraction to the condensate. The possibility of optical control on the quantum statistics of the output matter wave was pointed out in the framework of nonlinear atomic optical optics [12]. Several dynamical analysis concerning the nonclassical properties of the output of the trapped condensed atoms have been presented in Refs.[13,14,15].

On the other hand, there has been increased interest in the problem about decoherence of BEC and atom laser [16-19]. One fact causing decoherence comes from the nonlinear interaction between atoms. For a single-mode condensate the significant effect of atomic collisions is to cause fluctuations in the energy and hence fluctuations in the frequency, thus causing increased phase uncertainty, eventually lead to decoherence [16]. In Ref.[19], the influence of the decoherence on quantum coherent atomic tunneling between two condensates is studied. It is shown that the decoherence leads to the decay of the population difference and the suppression of the coherent atomic tunneling. In this paper, we would like to investigate the influence of decoherence on the nonclassical properties of the output of a Bose-Einstein condensate by adopting Milburn's model of intrinsic decoherence [20]. The influence of decoherence on the nonclassical properties, such as sub-poisson distribution and quadrature squeezing of the atom laser beam is investigated. It is shown that, under very special conditions, the atom laser beam may exhibit stationary quadrature squeezing in such a decoherence model.

The paper is organized as follows: In Sec.II we briefly outline the simple model describing the output coupling of the trapped dilute condensed atoms, in which the nonlinear interaction between the atoms and the quantized motion of atom center of mass in the inhomogeneous magnetic field has been ignored. Then, the quantum dynamical behavior of this system in the intrinsic decoherence model is discussed by making use of Bogoliubov approximation. In Sec.III, the influence of decoherence on the nonclassical properties, such as sub-poisson distribution, quadrature squeezing effect is investigated. In Sec.IV, there are some discussions.

II. THE MODEL

In this section, we consider the output coupling of the trapped dilute condensed atoms. For simplicity, the atoms are assumed to have two states, $|T\rangle$ and $|F\rangle$, with the initial condensation occurring in the trapped state $|T\rangle$. State $|F\rangle$, which is typically unconfined by the magnetic trap, is coupled to $|T\rangle$ by a one-mode squeezed optical field tuned near resonance with the $|T\rangle \rightarrow |F\rangle$ transition. The Hamiltonian of this system is given as [14,15] ($\hbar = 1$)

$$H = \omega_0 b^\dagger b + \omega_a a^\dagger a + \Omega(ab^\dagger c + a^\dagger bc^\dagger), \tag{1}$$

where $b(b^\dagger)$ and $c(c^\dagger)$ are the annihilation (creation) operators of bosonic atoms for the untrapped state $|F\rangle$ and the trapped state $|T\rangle$ with transition frequency ω_0, respectively. $a(a^\dagger)$ are the annihilation (creation) operators of the optical field with frequency ω_a. Here $\Omega = \sqrt{\omega_a/2\varepsilon_0 V}$ is the coupling constant, and V is the effective mode volume and ε_0 the vacuum permittivity. In the system (1), the atom-atom coupling and the nonlinear interaction between the atoms and the quantization motion of atomic center of mass in the trapped state by an inhomogeneous magnetic field has been ignored. It was shown that the above system leads to an oscillation behavior of the quantum statistics between the optical field and the output atomic laser beam [15].

In what follows, we outline the basic content of the Milburn model of decoherence. Based on an assumption that on sufficiently short time steps the quantum system does not evolves continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations, Milburn has derived the equation for the time evolution density operator $\rho(t)$ of the quantum system [20],

$$\frac{d\rho(t)}{dt} = \gamma[\exp(-\frac{i}{\gamma}H)\rho(t)\exp(\frac{i}{\gamma}H) - \rho(t)], \tag{2}$$

where γ is the mean frequency of the unitary time step. In the limit $\gamma \rightarrow \infty$, Eq.(2) reduces to the ordinary von Neuman equation for the density operator. It is easy to obtain the formal solution of Eq.(2) as follows,

$$\rho(t) = \sum_{k=0}^{\infty} A_k(t)\rho(0)A_k^\dagger(t), \tag{3}$$

where the Kraus operator $A_k(t)$ is given by

$$A_k(t) = \frac{(\gamma t)^{k/2}}{\sqrt{k!}} e^{-\gamma t/2} \exp(-i\frac{kH}{\gamma}). \tag{4}$$

Obviously, $\sum_{k=0}^{\infty} A_k^\dagger(t)A_k(t) = I$.

We assume that the initial state of system (1) is described by $\rho(0) = |\Psi(0)\rangle\langle\Psi(0)|$. Here, $|\Psi(0)\rangle = |\alpha\rangle_T \otimes |\Phi(0)\rangle_s$ with $|\alpha\rangle_T$ the Glauber coherent state of the operator c characterizing the condensed atoms in the trapped state $|T\rangle$; $|\Phi(0)\rangle_s = |0\rangle_F \otimes |\xi\rangle$, $|0\rangle_F$ represents that there is initially no occupying atoms in the untrapped state $|F\rangle$, and

the optical field is in the squeezed vacuum state $|\xi\rangle = S(\xi)|0\rangle$. $S(\xi) = \exp(\xi a^{\dagger 2} - \xi^* a^2)$ is the squeezed operator.
Substituting the $\rho(0)$ into Eq.(3), we obtain

$$\rho(t) = e^{-\gamma t}\exp(\alpha c^\dagger - \alpha^* c)\sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}M^k|0\rangle_{TT}\langle 0|\otimes|\Phi(0)\rangle_{ss}\langle\Phi(0)|M^{\dagger k}\exp(-\alpha c^\dagger + \alpha^* c) \tag{5}$$

where

$$M^k = \exp[-i\frac{k}{\gamma}(H_0 + H_1)], \tag{6}$$

$$H_0 = \omega_0 b^\dagger b + \omega_a a^\dagger a + \Omega(\alpha a b^\dagger + \alpha^* a^\dagger b), \tag{7}$$

$$H_1 = \Omega(a c b^\dagger + a^\dagger c^\dagger b). \tag{8}$$

If $|\alpha| \gg 1$, we can adopt the Bogoliubov approximation [21], i.e., neglect H_1 in Eq.(5). Then in the Bogoliubov approximation, the density operator $\rho(t)$ can be reexpressed as follows,

$$\rho(t) = |\alpha\rangle_{TT}\langle\alpha|\otimes\rho_s(t), \tag{9}$$

where

$$\rho_s(t) = e^{-\gamma t}\sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}\exp(-i\frac{kH_0}{\gamma})|0\rangle_{FF}\langle 0|\otimes|\xi\rangle\langle\xi|\exp(i\frac{kH_0}{\gamma}), \tag{10}$$

is the reduced density operator describing the subsystem of the untrapped atoms and optical field.

Now, we confine ourselves in the resonance case, i.e., $\omega_0 = \omega_a = \omega$. We define the operators $a(k)$ ($a^\dagger(k)$) and $b(k)$ ($b^\dagger(k)$) as

$$\begin{aligned}
a(k) &= \exp(\frac{ikH_0}{\gamma})a\exp(-\frac{ikH_0}{\gamma}),\\
a^\dagger(k) &= \exp(\frac{ikH_0}{\gamma})a^\dagger\exp(-\frac{ikH_0}{\gamma}),\\
b(k) &= \exp(\frac{ikH_0}{\gamma})b\exp(-\frac{ikH_0}{\gamma}),\\
b^\dagger(k) &= \exp(\frac{ikH_0}{\gamma})b^\dagger\exp(-\frac{ikH_0}{\gamma}).
\end{aligned} \tag{11}$$

It is easy to obtain that

$$\begin{aligned}
a(k) &= \cos(\frac{k\Omega'}{\gamma})e^{-ik\omega/\gamma}a - i\sin(\frac{k\Omega'}{\gamma})e^{i(\theta - k\omega/\gamma)}b,\\
a^\dagger(k) &= \cos(\frac{k\Omega'}{\gamma})e^{ik\omega/\gamma}a^\dagger + i\sin(\frac{k\Omega'}{\gamma})e^{-i(\theta - k\omega/\gamma)}b^\dagger,\\
b(k) &= \cos(\frac{k\Omega'}{\gamma})e^{-ik\omega/\gamma}b - i\sin(\frac{k\Omega'}{\gamma})e^{-i(\theta + k\omega/\gamma)}a,
\end{aligned}$$

$$b^\dagger(k) = \cos(\frac{k\Omega'}{\gamma})e^{ik\omega/\gamma}b^\dagger + i\sin(\frac{k\Omega'}{\gamma})e^{i(\theta+k\omega/\gamma)}a^\dagger, \tag{12}$$

where $\Omega' = |\alpha|\Omega$ and $e^{-i\theta} = \alpha/|\alpha|$. By making use of Eq.(12), we can express average values of arbitrary operator functionals $G(a, a^\dagger; b, b^\dagger)$ as following

$$\mathrm{Tr}[G(a, a^\dagger; b, b^\dagger)\rho_s(t)] = e^{-\gamma t}\sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}\mathrm{Tr}[G(a(k), a^\dagger(k); b(k), b^\dagger(k))|0\rangle_{FF}\langle 0| \otimes |\xi\rangle\langle\xi|]. \tag{13}$$

III. THE INFLUENCE OF INTRINSIC DECOHERENCE ON THE NONCLASSICAL PROPERTIES OF THE ATOM LASER BEAM

In this section, we investigate the nonclassical properties of the atom laser in the Milburn's model of decoherence. The average numbers and the fluctuation of the output atoms as well as the out-state photon can be obtained by making use of the Eq.(13)

$$N_a(t) = \mathrm{Tr}(a^\dagger a\rho_s(t))$$
$$= [\frac{1}{2} + \frac{1}{4}\exp(\gamma t e^{2i\Omega'/\gamma} - \gamma t) + \frac{1}{4}\exp(\gamma t e^{-2i\Omega'/\gamma} - \gamma t)]\sinh^2 r, \tag{14}$$

$$N_b(t) = \mathrm{Tr}(b^\dagger b\rho_s(t))$$
$$= [\frac{1}{2} - \frac{1}{4}\exp(\gamma t e^{2i\Omega'/\gamma} - \gamma t) - \frac{1}{4}\exp(\gamma t e^{-2i\Omega'/\gamma} - \gamma t)]\sinh^2 r, \tag{15}$$

$$\triangle N_a^2(t) = \mathrm{Tr}(a^\dagger a a^\dagger a\rho_s(t)) - [\mathrm{Tr}(a^\dagger a\rho_s(t))]^2$$
$$= \{\frac{3}{4} + e^{-\gamma t}[\frac{1}{2}\exp(\gamma t e^{2i\Omega'/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i\Omega'/\gamma}) + \frac{1}{8}\exp(\gamma t e^{4i\Omega'/\gamma}) + \frac{1}{8}\exp(\gamma t e^{-4i\Omega'/\gamma})]\}\sinh^2 r\cosh^2 r$$
$$+\{\frac{1}{8} + \frac{1}{16}e^{-\gamma t}[\exp(\gamma t e^{4i\Omega'/\gamma}) + \exp(\gamma t e^{-4i\Omega'/\gamma})]$$
$$-\frac{1}{16}e^{-2\gamma t}[\exp(2\gamma t e^{2i\Omega'/\gamma}) + \exp(2\gamma t e^{-2i\Omega'/\gamma}) + 2\exp(2\gamma t\cos(2\Omega'/\gamma))]\}\sinh^4 r$$
$$+\{\frac{1}{8} - \frac{1}{16}e^{-\gamma t}[\exp(\gamma t e^{4i\Omega'/\gamma}) + \exp(\gamma t e^{-4i\Omega'/\gamma})]\}\sinh^2 r, \tag{16}$$

$$\triangle N_b^2(t) = \mathrm{Tr}(b^\dagger b b^\dagger b\rho_s(t)) - [\mathrm{Tr}(b^\dagger b\rho_s(t))]^2$$
$$= \{\frac{3}{4} - e^{-\gamma t}[\frac{1}{2}\exp(\gamma t e^{2i\Omega'/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i\Omega'/\gamma}) - \frac{1}{8}\exp(\gamma t e^{4i\Omega'/\gamma}) - \frac{1}{8}\exp(\gamma t e^{-4i\Omega'/\gamma})]\}\sinh^2 r\cosh^2 r$$
$$+\{\frac{1}{8} + \frac{1}{16}e^{-\gamma t}[\exp(\gamma t e^{4i\Omega'/\gamma}) + \exp(\gamma t e^{-4i\Omega'/\gamma})]$$
$$-\frac{1}{16}e^{-2\gamma t}[\exp(2\gamma t e^{2i\Omega'/\gamma}) + \exp(2\gamma t e^{-2i\Omega'/\gamma}) + 2\exp(2\gamma t\cos(2\Omega'/\gamma))]\}\sinh^4 r$$
$$+\{\frac{1}{8} - \frac{1}{16}e^{-\gamma t}[\exp(\gamma t e^{4i\Omega'/\gamma}) + \exp(\gamma t e^{-4i\Omega'/\gamma})]\}\sinh^2 r, \tag{17}$$

where $r = 2|\xi|$.

In order to discuss the quantum statistical properties of the atom laser and the out-state optical field, we can calculate the Mandel Q parameters defined as [22]

$$Q_i(t) = \frac{\triangle N_i^2(t) - N_i(t)}{N_i(t)}, \quad (i = a, b) \tag{18}$$

$Q_i(t) < 0$, $Q_i(t) = 0$ or $Q_i(t) > 0$ mean the quantum state of the optical field or the atom laser field satisfy sub-Poisson, Poisson or super-Poisson distribution, respectively. In Fig.1 and Fig.2, the Mandel Q parameters Q_a and Q_b are plotted as a function of the time t for two different values of the parameter γ, respectively. We can observe that both the optical field and the atom laser beam satisfy the super-poisson distribution for any $t > 0$ in the cases with finite values of the parameter γ.

We now turn to discuss the influence of intrinsic decoherence on quadrature squeezing of atom laser and the optical field. We introduce four quadrature operators defined by

$$X_1^{(a)} = \frac{1}{2}(a + a^\dagger), \quad X_2^{(a)} = \frac{1}{2i}(a - a^\dagger), \quad X_1^{(b)} = \frac{1}{2}(b + b^\dagger), \quad X_2^{(b)} = \frac{1}{2i}(b - b^\dagger) \tag{19}$$

These operators satisfy the commutation relation

$$[X_1^{(a)}, X_2^{(a)}] = \frac{i}{2}, \quad [X_1^{(b)}, X_2^{(b)}] = \frac{i}{2}, \tag{20}$$

which implies the Heisenberg uncertainly relations

$$\langle(\triangle X_1^{(a)})^2\rangle\langle(\triangle X_2^{(a)})^2\rangle \geq \frac{1}{16}, \quad \langle(\triangle X_1^{(b)})^2\rangle\langle(\triangle X_2^{(b)})^2\rangle \geq \frac{1}{16}. \tag{21}$$

Squeezing is said to exist whenever $\langle(\triangle X_i^{(j)})^2\rangle < 1/4$, $(i = 1, 2)$, $(j = a, b)$. In order to characterize the influence of intrinsic decoherence on the quadrature squeezing of the atom laser and optical field, we calculate the following squeezing coefficients [23]

$$S_i^{(j)} = \frac{\langle(\triangle X_i^{(j)})^2\rangle - 0.25}{0.25}, \tag{22}$$

where $-1 \leq S_i^{(j)} < 0$ for quadrature squeezing. We express the squeezing coefficients as follows

$$S_1^{(a)} = 2N_a(t) + \mathrm{Re}\{e^{-\gamma t}[\exp(\gamma t e^{-2i\omega/\gamma}) + \frac{1}{2}\exp(\gamma t e^{2i(\Omega'-\omega)/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i(\Omega'+\omega)/\gamma})] \sinh r \cosh r e^{i\phi}\}, \tag{23}$$

$$S_2^{(a)} = 2N_a(t) - \mathrm{Re}\{e^{-\gamma t}[\exp(\gamma t e^{-2i\omega/\gamma}) + \frac{1}{2}\exp(\gamma t e^{2i(\Omega'-\omega)/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i(\Omega'+\omega)/\gamma})] \sinh r \cosh r e^{i\phi}\}, \tag{24}$$

$$\begin{aligned} S_1^{(b)} &= 2N_b(t) + \mathrm{Re}\{e^{-\gamma t - 2i\theta}[-\exp(\gamma t e^{-2i\omega/\gamma}) \\ &+ \frac{1}{2}\exp(\gamma t e^{2i(\Omega'-\omega)/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i(\Omega'+\omega)/\gamma})] \sinh r \cosh r e^{i\phi}\}, \end{aligned} \tag{25}$$

$$S_2^{(b)} = 2N_b(t) - \mathrm{Re}\{e^{-\gamma t - 2i\theta}[-\exp(\gamma t e^{-2i\omega/\gamma})$$
$$+ \frac{1}{2}\exp(\gamma t e^{2i(\Omega'-\omega)/\gamma}) + \frac{1}{2}\exp(\gamma t e^{-2i(\Omega'+\omega)/\gamma})]\sinh r\cosh r e^{i\phi}\}, \qquad (26)$$

where $e^{i\phi} = \xi/|\xi|$. It is obvious that $S_1^{(b)} + S_2^{(b)} = 4N_b(t)$, and $N_b(t)$ is always non-negative. So, we need only investigate $S_1^{(b)}$ or $S_2^{(b)}$ to explore the squeezing property of the atom laser. In what follows, it is assumed $\phi = \theta = 0$. We start our analysis of the squeezing properties of both the atom laser and the optical field in the limit case with $\gamma \to \infty$ and finite values of ω and Ω', which means not any decoherence is presented. Then, the Eqs.(23-26) reduces to the results in Ref.[15], in which the squeezing coefficients of both the atom laser and the optical field exhibit the Rabi-like oscillation. If γ is a finite value but remains large, i.e. $\omega/\gamma \ll 1$ and $\Omega'/\gamma \ll 1$, the expressions of the squeezing coefficients $S_i^{(a)}$ and $S_i^{(b)}$ $(i = 1, 2)$ can be approximated as

$$S_1^{(a)} \approx [1 + \cos 2\Omega' t \exp(-2\Omega'^2 t/\gamma)]\sinh^2 r + \{\cos 2\omega t \exp(-2\omega^2 t/\gamma)$$
$$+\frac{1}{2}\cos[2(\Omega'-\omega)t]\exp[-2(\Omega'-\omega)^2 t/\gamma]+\frac{1}{2}\cos[2(\Omega'+\omega)t]\exp[-2(\Omega'+\omega)^2 t/\gamma]\}\sinh r\cosh r, \qquad (27)$$

$$S_2^{(a)} \approx [1 + \cos 2\Omega' t \exp(-2\Omega'^2 t/\gamma)]\sinh^2 r - \{\cos 2\omega t \exp(-2\omega^2 t/\gamma)$$
$$+\frac{1}{2}\cos[2(\Omega'-\omega)t]\exp[-2(\Omega'-\omega)^2 t/\gamma]+\frac{1}{2}\cos[2(\Omega'+\omega)t]\exp[-2(\Omega'+\omega)^2 t/\gamma]\}\sinh r\cosh r, \qquad (28)$$

$$S_1^{(b)} \approx [1 - \cos 2\Omega' t \exp(-2\Omega'^2 t/\gamma)]\sinh^2 r + \{-\cos 2\omega t \exp(-2\omega^2 t/\gamma)$$
$$+\frac{1}{2}\cos[2(\Omega'-\omega)t]\exp[-2(\Omega'-\omega)^2 t/\gamma]+\frac{1}{2}\cos[2(\Omega'+\omega)t]\exp[-2(\Omega'+\omega)^2 t/\gamma]\}\sinh r\cosh r, \qquad (29)$$

$$S_2^{(b)} \approx [1 - \cos 2\Omega' t \exp(-2\Omega'^2 t/\gamma)]\sinh^2 r - \{-\cos 2\omega t \exp(-2\omega^2 t/\gamma)$$
$$+\frac{1}{2}\cos[2(\Omega'-\omega)t]\exp[-2(\Omega'-\omega)^2 t/\gamma]+\frac{1}{2}\cos[2(\Omega'+\omega)t]\exp[-2(\Omega'+\omega)^2 t/\gamma]\}\sinh r\cosh r, \qquad (30)$$

From Eqs.(27-30), we find that all of the squeezing coefficients $S_i^{(a)}$ and $S_i^{(b)}$ $(i = 1, 2)$ tend to a fixed positive value $\sinh^2 r$ as the time $t \to \infty$, except that they tend to $\sinh^2 r \pm \frac{1}{2}\sinh r\cosh r$ in the special case with $\Omega' = \omega$. With the further decrease of γ, the oscillatory behaviors of the squeezing properties of both the atom laser and the optical field become frozen. In Fig.3, the squeezing coefficient $S_2^{(b)}$ is plotted as a function of time t for three different values of parameter γ. With the decreases of parameter γ, we can observe rapid deterioration of the Rabi-like oscillation of squeezing coefficient. In Fig.3(b) and Fig3(c), if the time t become very large, the squeezing coefficient $S_2^{(b)}$ tends to be a fixed positive value 0.093, which means the decoherence eventually completely destroy the squeezing property of the atom laser. However, under very special conditions that $\Omega' = \omega \ll \gamma$, $0 < \tanh r < \frac{1}{2}$ and γ is large but remain finite, $S_2^{(b)}$ will tend to be a negative value as the time t approaches to infinite, which means the stationary state of the atom laser gets squeezed. This can be clearly seen from Fig.4.

Recently, the sensitivity of quantum systems that are chaotic in a classical limit to small perturbations has been investigated in Ref.[24], and the relation between the sensitive and decoherence has been discussed. In what follows, we briefly investigate influence of small perturbation on the quadrature squeezing of atom laser. From Eq.(26), we can observe that the dynamical behavior of $S_2^{(b)}$ is dependent on the values of the exponential facts $\gamma[e^{2i(\Omega'-\omega)/\gamma}-1]$, $\gamma[e^{\pm 2i\Omega'/\gamma}-1]$, $\gamma[e^{-2i\omega/\gamma}-1]$, and $\gamma[e^{-2i(\Omega'+\omega)/\gamma}-1]$. In the case with $\Omega' \simeq \omega \ll \gamma$, the term $\gamma[e^{2i(\Omega'-\omega)/\gamma}-1] \simeq (2i\delta - 2\delta^2/\gamma)$ plays a dominantly role in the long time dynamical behavior, where $\delta = \Omega' - \omega$. In Fig.5, we plotted $S_2^{(b)}$ as the function of time t for different small values of δ. Since Ω' is dependent on the amplitude α of the initial coherent state of condensed atoms in the trapped state $|T\rangle$, we expect that the squeezing dynamical behavior of the atom laser is sensitive to the initial state of trapped condensed atoms, just as shown in Fig.5.

IV. DISCUSSION

In this paper, we investigate nonclassical properties of the output of a Bose-Einstein condensate in Milburn's model of intrinsic decoherence by making use of Bogoliubov approximation. It is shown that the squeezing properties of the atom laser field can be destroyed by the decoherence under most conditions. However, under some very special conditions, the squeezing properties of atom laser is robust against the decoherence. This phenomenon is highly dependent on the particular modelling of decoherence.

In what follows, we briefly discuss the relevance of our theoretical results to the realistic experimental conditions. By making use of the preliminary atom laser experiments [8], as a guide to realistic parameter values, we choose $\omega = 300$kHz, $\Omega = 60$kHz, the initial trapped atom number $N = |\alpha|^2 = 10^6$, the mean frequency of the unitary time step $\gamma = 10^5$MHz, and $r = 1$ as a example to illustrate our theoretical results. In such a condition, the Rabi-like oscillation behaviors of the numbers of both the output atom and output photon become frozen after $100\mu s$. However, the atom laser persists in exhibiting oscillation of squeezing coefficient and its squeezing property is finally destroyed by decoherence till about $0.15s$.

Milburn's model of intrinsic decoherence is currently a very active field of research. Nevertheless, so far little attention has been paid to its study in the context of ultracold quantum gases. However, due to their high sensitivity to decoherence, such systems could provide an interesting testing ground for the Milburn model. In this paper, we provide a first step in this direction by studying the influence of intrinsic decoherence on the output properties of a simple atom laser model. A thorough discussion of all other sources of decoherence that could be present in an experiment and comparison with the results presented in this work should be interesting and necessary in the future study. It is also very interesting to investigate the entanglement between output atoms and output photons and discuss the influence of intrinsic decoherence on entanglement [25-28]. Moreover, it is worth to discuss the possible quantum chaotic dynamical behavior

of atom laser and the quantum-classical corresponding by fully taking account of the atom-atom interaction.

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References

[1] M. H. Anderson *et al.*, Science **269**, 198 (1995).

[2] K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).

[3] C. C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995).

[4] M. Olshanii, Y. Castin, and J. Dalibard, in *Proceedings of the XII Conference on Lasers Spectroscopy*, edited by M. Inguscio, M. Allegrini, and A. Sasso (World Scientific, Singapore, 1995).

[5] R. J. C. Spreeuw, T. Pfau, U. Janicke, and M. Wilkens, Europhys. Lett. **32**, 469 (1995).

[6] H. M. Wiseman and M. J. Collett, Phys. Lett. A **202**, 246 (1995).

[7] A. M. Guzman, M. Moore, and P. Meystre, Phys. Rev. A **53**, 977 (1996).

[8] M.-O Mewes *et al.*, Phys. Rev. Lett. **78**, 582 (1997).

[9] I. Bloch, T. W. Hánsch, and T. Esslinger, Phys. Rev. Lett. **82**, 3008 (1999).

[10] E. W. Hagley *et al.*, Science **283**, 1706 (1999).

[11] L. Deng *et al.*, Nature **398**, 218 (1999).

[12] M. G. Moore, O. Zobay and P. Meystre, e-print cond-mat/9902293.

[13] N. M. Bogoliubov, *et al.*, Phys. Rev. Lett. **86**, 4439 (2001).

[14] C. Sun, H. Zhan, Y. Miao, J. Li, Commun. Theor. Phys. **29**, 161 (1998).

[15] H. Jing, J.-L. Chen, M.-L. Ge, Phys. Rev. A **63**, 015601 (2001).

[16] L. K. Thomsen and H. M. Wiseman, Phys. Rev. A **65**, 063607 (2002).

[17] I. Bouchoule and K. Mølmer, Phys. Rev. A **67**, 011603(R) (2003).

[18] L. Pitaevskii and S. Stringari, Phys. Rev. Lett. **87**, 180402 (2001).

[19] L.-M. Kuang *et al.*, Phys. Rev. A **61**, 013608 (1999).

[20] G. J. Milburn, Phys. Rev. A **44**, 5401 (1991).

[21] F. Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463 (1999).

[22] D. F. Walls and G. J. Milburn, *Quantum Optics*, (Springer-Verlag, Berlin, 1994).

[23] V. Buzek, A. V. Barranco and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).

[24] Z. P. Karkuszewski, C. Jarzynski, and W. H. Zurek, Phys. Rev. Lett. **89**, 170405 (2002).

[25] S.-B. Li, J.-B. Xu, Phys. Lett. A **311**, 313 (2003).

[26] S.-B. Li, J.-B. Xu, Phys. Lett. A **313**, 175 (2003).

[27] S.-B. Li, J.-B. Xu, Chin. Phys. Lett. **20**, 985 (2003).

[28] W.-J. Li, S.-B. Li and J.-B. Xu, Chin. Phys. Lett. **21**, 774 (2004).

Figure Caption

FIG.1. The Mandel Q parameter Q_a of optical field is plotted as a function of time t with $\Omega' = 1$ and $r = 2$ for two different values of the parameter γ, (Solid line) $\gamma = \infty$, (Dot line) $\gamma = 10^2$.

FIG.2. The Mandel Q parameter Q_b of atom laser field is plotted as a function of time t with $\Omega' = 1$ and $r = 2$ for two different values of the parameter γ, (Solid line) $\gamma = \infty$, (Dot line) $\gamma = 10^2$.

FIG.3. The quadrature squeezing coefficient $S_2^{(b)}$ of the atom laser field is plotted as a function of time t with $\omega = 0.1$, $\phi = 0$, $\theta = 0$, $\Omega' = \pi$ and $r = 0.3$ for three different values of the parameter γ, (a) $\gamma = \infty$, (b) $\gamma = 10^3$, (c) $\gamma = 10^2$.

FIG.4. The quadrature squeezing coefficient $S_2^{(b)}$ of the atom laser field is plotted as a function of time t with $\omega = 10$, $\phi = 0$, $\theta = 0$, $\Omega' = 10$, $r = 0.3$ and $\gamma = 10^2$.

FIG.5. The quadrature squeezing coefficient $S_2^{(b)}$ of the atom laser field is plotted as a function of time t with $\omega = 10$, $\phi = 0$, $\theta = 0$, $r = 0.4$ and $\gamma = 10^2$ for four different values of Ω': (Solid line) $\Omega' = 10$, (Dot line) $\Omega' = 10 + 10^{-7}$, (Dash Dot line) $\Omega' = 10 + 2 \times 10^{-7}$, (Dash Dot Dot line) $\Omega' = 10 + 3 \times 10^{-7}$.

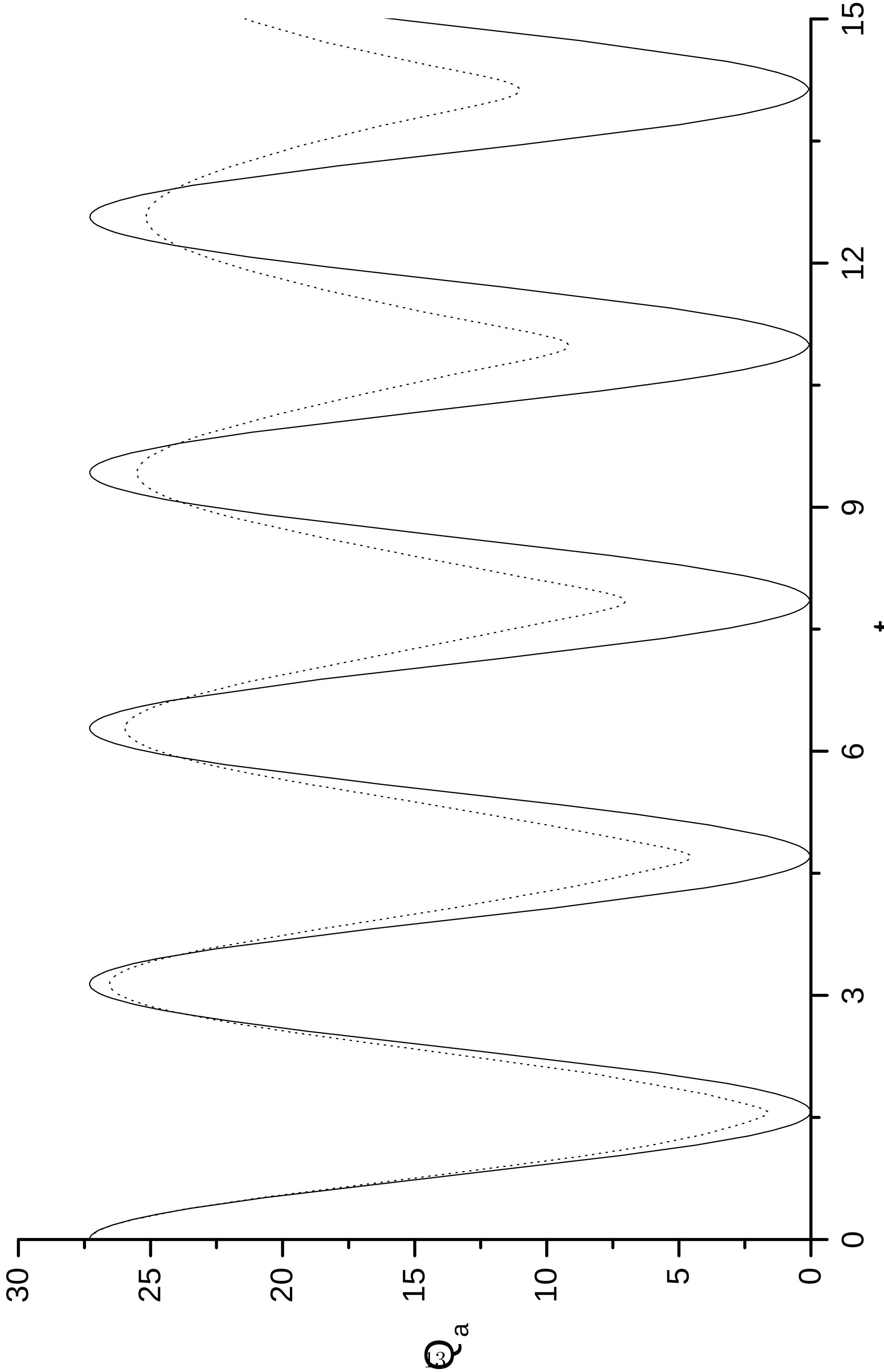


Figure 1: The Mandel Q parameter Q_a of optical field is plotted as a function of time t with $Q' = 1$ and $r = 2$ for two different values of the parameter γ. (Solid line) $\gamma = \infty$

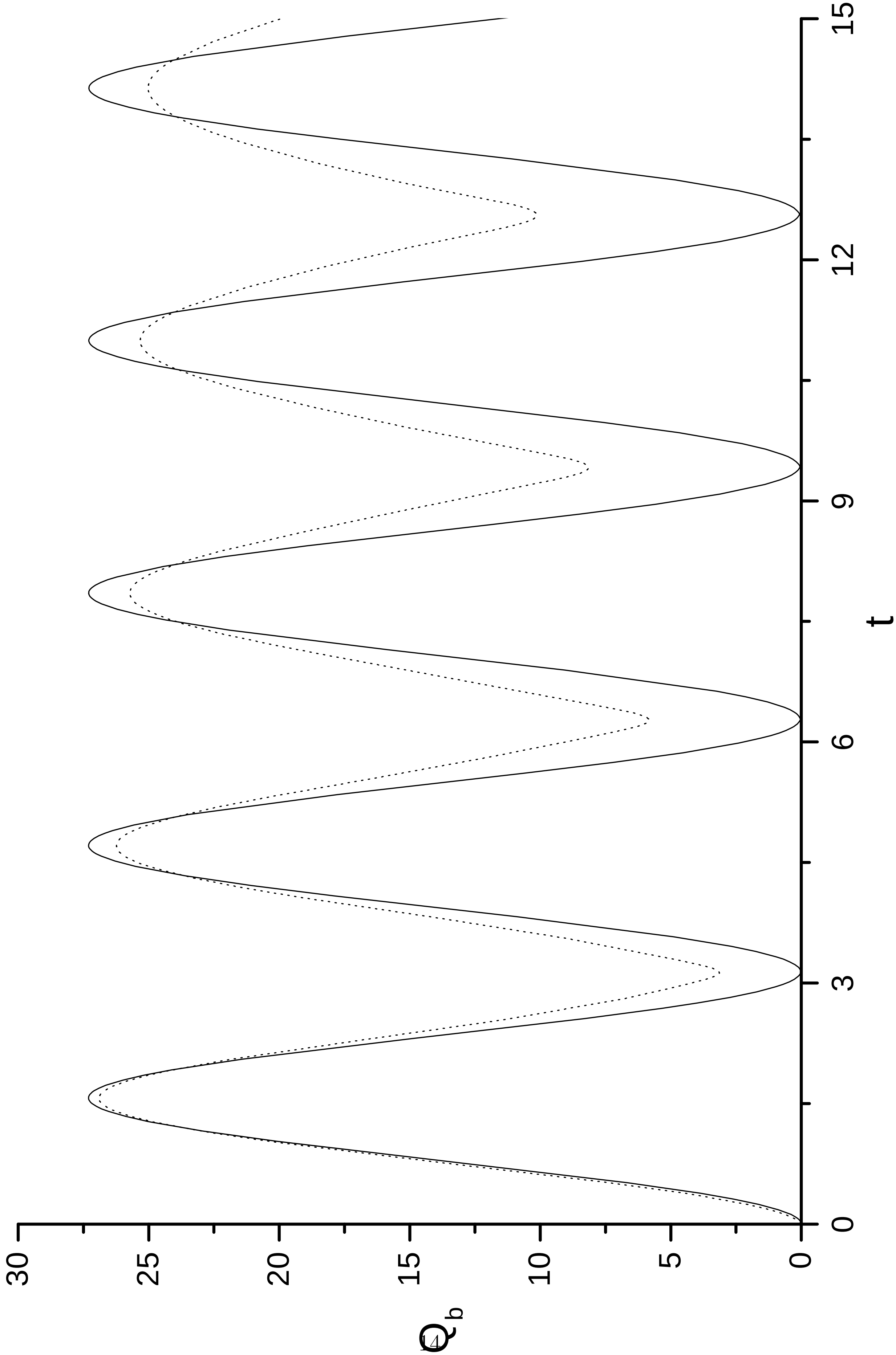


Figure 2: The Mandel Q parameter Q_b of atom laser field is plotted as a function of time t with $\Omega' = 1$ and $r = 2$ for two different values of the parameter γ. (Solid line)

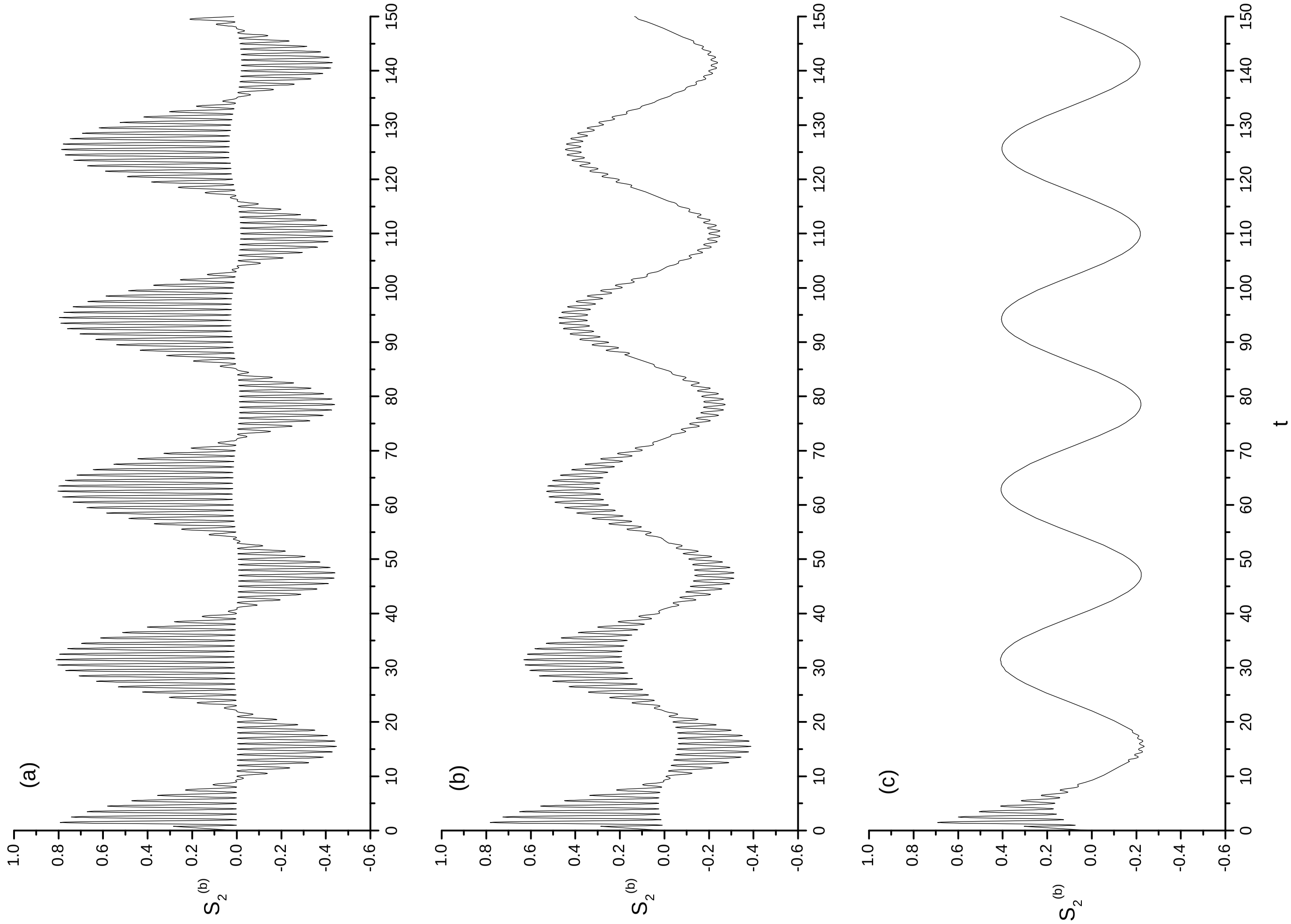
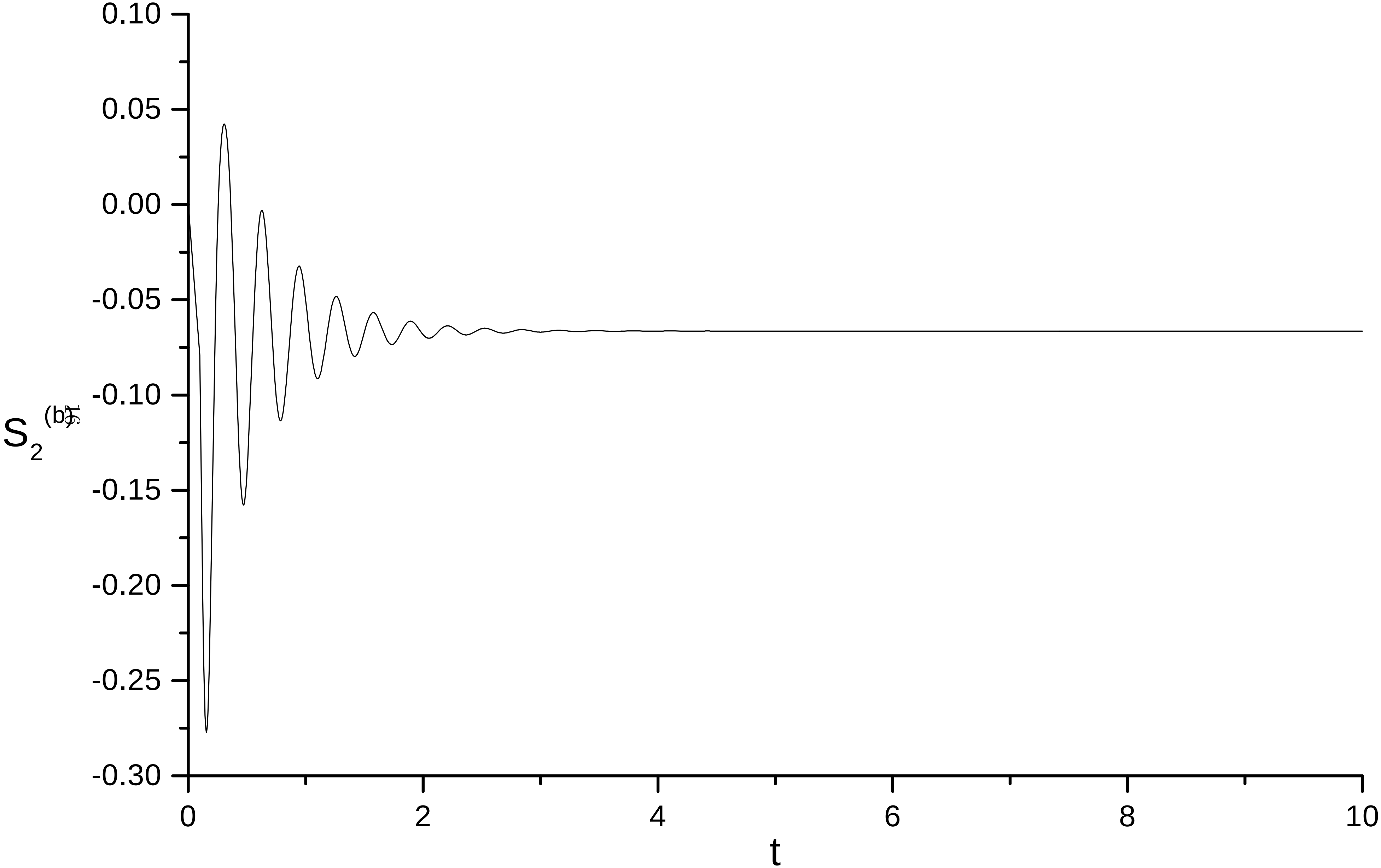
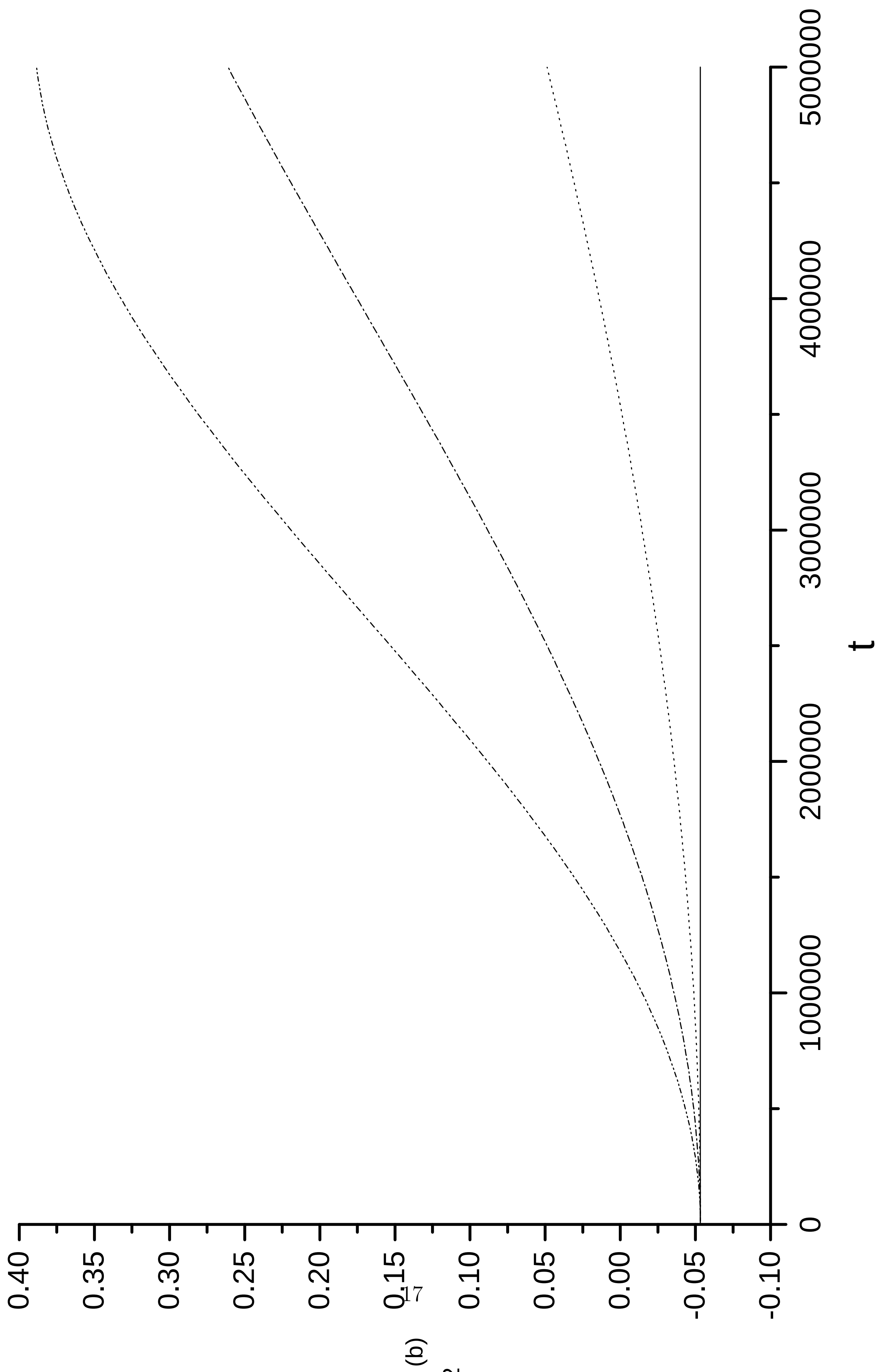


Figure 3: The quadrature squeezing coefficient $S_2^{(b)}$ of the atom laser field is plotted as a function of time t with $\omega = 0.1$, $\phi = 0$, $\theta = 0$, $\Omega' = \pi$ and $r = 0.3$ for three different values of the parameter γ, (a) $\gamma = \infty$, (b) $\gamma = 10^3$, (c) $\gamma = 10^2$.



0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
$S_2^{(b)}$
0
2
4
6
8
10
t


(b)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
0
1000000
2000000
3000000
4000000
5000000
t